EFFECT OF THE INTERACTION OF GASES WITH A SURFACE ON THERMAL CREEP IN A FLAT CHANNEL

V. G. Chernyak, A. E. Margilevskii,
B. T. Porodnov, and P. E. Suetin

UDC 533.6.011.8

The dependence of the thermal creep in a flat channel on the coefficient of accommodation of tangential momentum with an arbitrary mode of gas flow is studied. The problem is solved on the basis of a linearized kinetic equation using the BGK (Bhatnager–Gross–Krook) model for the molecular collision operator. The Maxwell specular-diffuse system is taken as the boundary conditions.

As is known, the macroscopic movement of gas along an unevenly heated wall is called thermal creep [1]. It's value is determined by the Knudsen number (Kn), the geometry of the channel, and the nature of the interaction of the gas molecules with the surface over which the flow occurs.

Many reports devoted to the theoretical study of this effect in a wide range of Knudsen numbers have been published in recent times. As a rule, the problem has been solved on the basis of the linearized Boltzmann equation or statistical models of it. The most varied methods of solving the kinetic equations have been used for this: the moment method [2, 3], the method of iterations of the moment solutions [4, 5], the Monte Carlo method [6], and finally, integral methods [7-12]. In all these reports, however, completely diffuse scattering of the gas molecules by the channel walls was taken as the boundary conditions.

Recently the authors of the present report attempted to describe the effect of the thermomolecular pressure difference during the arbitrary accommodation of the tangential momentum of molecules incident on a wall [13]. In doing this it was assumed that the thermal creep is independent of the nature of the interaction of the gas molecules with the surface of the channel. Such an assumption is fully confirmed by theory [14] if one uses the boundary conditions proposed in [15]. As has been indicated earlier [16, 17], however, this approach does not take into account the dependence of the disturbance in the Maxwellian distribution function on the velocity of the molecules reflected by the wall.

In the present report we use stricter boundary conditions, proposed again by Maxwell, according to which a portion ε of the molecules incident on the wall are scattered diffusely from it while a portion $(1-\varepsilon)$ are reflected specularly. The parameter ε has also received the name of the coefficient of accommodation of tangential momentum. Of course, the Maxwellian boundary conditions are semiempirical in the sense that the parameter ε cannot be calculated theoretically. Nevertheless, its introduction into the theory proves very useful, firstly, because it helps to at least qualitatively estimate the effect of the wall on the phenomenon studied, and secondly, because it allows one from a comparison of theoretical and experimental data to extract such an important characteristic of the gas–solid interaction as the coefficient of accommodation of tangential momentum.

A procedure which the authors used earlier [18] in a study of plane Poiseuille flow is applied in the present work.

Let us consider a gas between infinite parallel plates located at $X = \pm d/2$ (d is the distance between the plates) whose state is disturbed by a longitudinal temperature gradient. Macroscopic movement of the gas occurs in this case, characterized by a velocity U_t and called thermal creep.

S. M. Kirov Ural Polytechnic Institute, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 28, No. 4, pp. 624-629, April, 1975. Original article submitted January 25, 1974.

The disturbance of the gas is assumed to be small enough that its state can be described by a distribution function which differs insignificantly from a Maxwellian distribution:

$$f(\vec{v}, X, z) = f_0(v, z)[1 + h(\vec{v}, X)],$$

$$f_0(v, z) = n(z)\left[\frac{m}{2\pi kT(z)}\right]^{3/2} \exp\left[-\frac{mv^2}{2kT(z)}\right], \tag{1}$$

where n and T are the numerical density and the temperature of the gas, respectively; m is the mass of a molecule; k is Boltzmann's constant; $\vec{v}$ is the proper velocity of a molecule and z is the longitudinal coordinate, which coincides with the axis of symmetry of the flat channel. In this case with allowance for (1) the fundamental kinetic equation is linearized and for the BGK (Bhatnager–Gross–Krook) model of the intermolecular collision operator [19] is written in the following form:

$$c_x \frac{\partial h}{\partial x} + c_z\left(c^2 - \frac{5}{2}\right)\tau = \delta\left[2c_z u_t(x) - h\right].$$

Here we introduce the dimensionless values

$$c_i = \left(\frac{m}{2kT_0}\right)^{1/2} v_i, \quad u_t = \left(\frac{m}{2kT_0}\right)^{1/2} U_t, \quad x = X/d,$$

$$\tau = \frac{d}{T}\frac{dT}{dz}, \quad \delta = \frac{\sqrt{\pi}}{2}\frac{d}{\lambda} = \frac{\sqrt{\pi}}{2}\mathrm{Kn}^{-1}, \tag{2}$$

where T_0 is the average temperature of the system; λ is the mean free path of the molecules, calculated from the viscosity coefficient for solid spherical molecules at the temperature T_0.

Further, one must allow for the discontinuous nature of the distribution function near the channel walls at $c_x = 0$, i.e.,

$$f = \begin{cases} f^+ & \text{for} \quad c_x > 0, \\ f^- & \text{for} \quad c_x < 0. \end{cases} \tag{3}$$

Then the Maxwellian boundary conditions for the distribution function take the from

$$f^{\pm}\left(\mp\frac{1}{2}, z, c_x, c_y, c_z\right) = \varepsilon f_0 + (1 - \varepsilon)\cdot f^{\mp}\left(\mp\frac{1}{2}, z, -c_x, c_y, c_z\right). \tag{4}$$

The boundary conditions for the disturbance function follow from this with allowance for the linearization of (1):

$$h^{\pm}\left(\mp\frac{1}{2}, c_x, c_z\right) = (1 - \varepsilon)h^{\mp}\left(\mp\frac{1}{2}, -c_x, c_z\right). \tag{5}$$

In addition, the symmetry of the problem requires that

$$h^{\pm}(\mp x, \vec{c}) = h^{\mp}(\pm x, \vec{c}). \tag{6}$$

Equation (2) can be formally integrated if one temporarily considers u_t to be a known function of the coordinates:

$$h^{\pm}(x, \vec{c}) = \exp\left(-\frac{\delta}{c_x}x\right)\left\{\exp\left(\mp\frac{\delta}{2c_x}\right)h^{\pm}\left(\mp\frac{1}{2}, \vec{c}\right) + \frac{c_z}{c_x}\int_{\mp 1/2}^{x}\left[2\delta u_t(s) - \tau\left(c^2 - \frac{5}{2}\right)\right]\exp\left(\frac{\delta}{c_x}s\right)ds\right\}. \tag{7}$$

The integration constants $h(\pm 1/2, \vec{c})$ in Eqs. (7) must be determined from the boundary conditions (5) and the conditions (6) of symmetry of the problem. As a result it is easy to obtain the following values of the sought constants:

$$h^{\pm}\left(\mp\frac{1}{2}, \vec{c}\right) = \frac{(1 - \varepsilon)\exp\left(-\frac{\delta}{2c_x}\right)}{1 - (1 - \varepsilon)\exp\left(\mp\frac{\delta}{c_x}\right)}\int_{\mp 1/2}^{\pm 1/2}\frac{c_z}{c_x}\left\{2\delta u_\tau(s)\right.$$

$$\left. - \tau\left(c^2 - \frac{5}{2}\right)\right\}\exp\left(\frac{\delta}{c_x}s\right)ds. \tag{8}$$

TABLE 1. Values of Thermal Creep Q_t for Different Values of the Parameters δ and ε

δ	ε					
	1	0,98	0,96	0,94	0,92	0,90
0,01	1,2358	1,2710	1,3074	1,3450	1,3840	1,4242
0,03	0,9635	0,9884	1,0140	1,0404	1,0676	1,0958
0,05	0,8452	0,8656	0,8866	0,9083	0,9306	0,9537
0,07	0,7707	0,7884	0,8066	0,8254	0,8447	0,8645
0,09	0,7170	0,7327	0,7489	0,7656	0,7828	0,8004
0,1	0,6949	0,7099	0,7253	0,7412	0,7574	0,7742
0,3	0,4842	0,4920	0,5000	0,5081	0,5165	0,5250
0,5	0,3985	0,4036	0,4089	0,4142	0,4197	0,4253
0,7	0,3463	0,3500	0,3537	0,3576	0,3615	0,3654
0,9	0,3096	0,3124	0,3151	0,3180	0,3209	0,3238
1	0,2948	0,2972	0,2996	0,3021	0,3046	0,3071
2	0,2060	0,2067	0,2073	0,2080	0,2086	0,2093
3	0,1615	0,1615	0,1616	0,1617	0,1618	0,1619
4	0,1334	0,1333	0,1331	0,1330	0,1328	0,1327
5	0,1138	0,1136	0,1133	0,1131	0,1128	0,1126
6	0,0993	0,0990	0,0987	0,0984	0,0981	0,0978
7	0,0880	0,0877	0,0874	0,0871	0,0867	0,0864
8	0,0790	0,0787	0,0784	0,0781	0,0777	0,0774
9	0,0716	0,0713	0,0710	0,0707	0,0704	0,0701
10	0,0655	0,0652	0,0649	0,0646	0,0643	0,0640

By definition the macroscopic velocity of the gas is given by the following expression:

$$u_t = \frac{1}{n}\int\limits_{c_x \lessgtr 0} c_z f^{\pm}(\vec{c}, x, z)\,\vec{dv} = \pi^{-3/2}\left[\int\limits_{c_x>0} c_z \exp(-c^2) h^+ \vec{dc} + \int\limits_{c_x<0} c_z \exp(-c^2) h^- \vec{dc}\right]. \tag{9}$$

Since the functions $h^\pm(x, \vec{c})$ are fully determined by Eqs. (7) and (8) there is no great difficulty in obtaining from (9) the integral transfer equation for the thermal creep velocity:

$$\Psi_t(x) = \varphi(x) + \frac{\delta}{\sqrt{\pi}}\int\limits_{-1/2}^{+1/2} K(x, s)\Psi_t(s)\,ds, \tag{10}$$

where

$$\Psi_t = \frac{1}{2} + \frac{2\delta}{\tau}u_t,\quad \varphi(x) = \frac{1}{2} - \frac{\delta}{\sqrt{\pi}}\int\limits_{-1/2}^{+1/2}\{I_1(\delta|x-s|) + (1-\varepsilon)[T_1(\delta, x-s+1) + T_1(\delta, s-x+1)]\}\,ds,$$

$$K(x, s) = I_{-1}(\delta|x-s|) + (1-\varepsilon)[T_{-1}(\delta, s-x+1) + T_{-1}(\delta, x-s+1)], \tag{11}$$

$$I_n(t) = \int\limits_0^\infty y^n \exp\left(-y^2 - \frac{t}{y}\right)dy,$$

$$T_n(u, v) = \int\limits_0^\infty \frac{y^n \exp\left(-y^2 - \frac{u}{y}v\right)}{1-(1-\varepsilon)\exp\left(-\frac{u}{y}\right)}\,dy.$$

It is easy to see that in the particular case of fully diffuse scattering of the molecules by the channel walls ($\varepsilon = 1$) Eq. (10) coincides with that presented in [9].

The Bubnov–Galerkin method [20] is used in the present work for the solution of Eq. (10).

Following the basic idea of this method, one must choose a system of base functions. From physical considerations it is convenient to choose even functions of the type $\{x^{2k}\}$ with $k = 0, 1, 2, \ldots$. The rapid convergence of the method used for the chosen base allows one to be confined to only two terms in the expansion of the unknown function Ψ_t:

$$\Psi_t \approx a_0 + a_1 x^2 + \ldots \tag{12}$$

As calculations show, taking into account the third term in the expansion $\Psi_t(x)$ corrects the result obtained by less than 1%.

Then one must substitute (12) into the original Eq. (10) and demand the orthogonality of the equation obtained to the base functions $[1, x^2]$. In this case the scalar product of any two functions F(x) and G(x) in the Hilbert space $L_2(-1/2, +1/2)$ is determined as

$$(F, G) = \int_{-1/2}^{+1/2} G(x) F(x)\, dx. \tag{13}$$

Thus, we obtain the following system of algebraic equations for the determination of the coefficients a_0 and a_1 of the expansion (12):

$$a_0 \alpha_{11} + a_1 \alpha_{12} = \beta_1, \quad a_0 \alpha_{21} + a_1 \alpha_{22} = \beta_2, \tag{14}$$

where

$$\alpha_{11} = (1, 1) - \frac{\delta}{\sqrt{\pi}} (1, \vec{K} 1), \; \alpha_{12} = \alpha_{21} = (1, x^2) - \frac{\delta}{\sqrt{\pi}} (1, \vec{K} s^2),$$

$$\alpha_{22} = (x^2, x^2) - \frac{\delta}{\sqrt{\pi}} (x^2, \vec{K} s^2), \; \beta_1 = (1, \varphi), \; \beta_2 = (x^2, \varphi). \tag{15}$$

Here $\vec{K}\Phi$ is an integral operator corresponding to the kernel K(x, s) of (11) and acting on the arbitrary function Φ.

Since the function $\Psi_t(x)$ is determined by Eqs. (12), (14), and (15), one can write an expression for the dimensionless gas flux characterizing the thermal creep in a flat channel:

$$Q_t = \frac{2}{\tau} \int_{-1/2}^{+1/2} u_t\, dx = -\frac{1}{\delta} \left(\frac{1}{2} - \int_{-1/2}^{+1/2} \Psi_t(x)\, dx \right) = -\frac{1}{\delta} \left[\frac{1}{2} - \frac{\beta_1 \alpha_{22} - \beta_2 \alpha_{12} + (\beta_2 \alpha_{11} - \beta_1 \alpha_{12})/12}{\alpha_{11} \alpha_{22} - \alpha_{12}^2} \right]. \tag{16}$$

In the limiting cases of an almost free-molecular mode of flow ($\delta \to 0$) and of a viscous mode of flow with slippage ($\delta \to \infty$) the general equation (16) is transformed into the following simple equations:

$$Q_t (\delta \to 0, \varepsilon) = -\frac{2 - \varepsilon}{\varepsilon} \frac{\ln \delta}{2\sqrt{\pi}}, \tag{17}$$

$$Q_t (\delta \to \infty, \varepsilon) = \frac{A_t}{\delta}, \text{ where } A_t = \frac{1}{2} + \frac{\varepsilon}{4}. \tag{18}$$

The coefficient A_t is called the thermal creep constant. The dependence of this constant on the coefficient of accommodation of tangential momentum was studied in [14] on the basis of the BGK model. As a result an exact expression of the type $A_t = 1/2 + 0.2662\ \varepsilon$, which agrees well with Eq. (18), was obtained by the method of "elementary solutions."

The results of the calculation of Q_t by Eq. (16) for intermediate values of the parameter δ of rarefaction of the gas and for certain values of the coefficient of accommodation of tangential momentum ε are presented in Table 1 (the calculations were performed on a BÉSM-4M computer).

As seen from the table, when $\delta \sim 3.5$ the thermal creep does not depend at all on the nature of the interaction of the molecules with the channel wall. At the same time, with an increase in the portion of the molecules specularly reflected from the wall it increases when $\delta < 3.5$ and decreases when $\delta > 3.5$.

Such a dependence of the thermal creep on the coefficient of accommodation of tangential momentum ε must be taken into account in a study of the effect of the thermomolecular pressure difference and thermophoresis.

NOTATION

U_t	is the velocity of thermal creep;
T	is the gas temperature;

m	is the mass of a molecule;
$\vec{v}$	is the proper velocity of a molecule;
$\tau = (d/T)dT/dz$	is the dimensionless temperature gradient;
$\delta = \sqrt{\pi}/(2\mathrm{Kn})$	is the parameter of gas rarefaction;
ε	is the coefficient of accommodation of tangential momentum;
Q_t	is the dimensionless flux of thermal creep averaged over channel cross section;
A_t	is the thermal creep constant.

LITERATURE CITED

1. J. C. Maxwell, Roy. Soc. London, Phil. Trans., 170, 230-256 (1879).
2. S. F. Borisov, B. T. Porodnov, and P. E. Suetin, Zh. Tekh. Fiz., 40, 501 (1970).
3. A. I. Karpenko, S. G. Skakun, P. E. Suetin, and V. G. Chernyak, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4, 182 (1973).
4. P. E. Suetin, S. G. Skakun, and V. G. Chernyak, Zh. Tekh. Fiz., 41, 1738 (1971).
5. P. E. Suetin, S. G. Skakun, and V. G. Chernyak, in: Diffusion in Gases and Liquids [in Russian], Alma-Ata (1972).
6. S. L. Gorelov and M. N. Kogan, Uch. Zap. Tsentr. Aéro-Gidrodinam. Inst., 1, No. 6 (1970).
7. Y. Sone and K. Yamamoto, Phys. Fluids, 11, 8 (1968).
8. S. K. Loyalka, Phys. Fluids, 12, 2301 (1969).
9. M. N. Kogan and N. K. Makashev, Uch. Zap. Tsentr. Aéro-Gidrodinam. Inst., 1, No. 2 (1970).
10. N. K. Makashev, Uch. Zap. Tsentr. Aero-Gidrodinam. Inst., 1, No. 6 (1970).
11. V. G. Chernyak and P. E. Suetin, in: Heat and Mass Transfer [in Russian], Vol. 8, Minsk (1972), p. 553.
12. V. G. Chernyak, B. T. Porodnov, and P. E. Suetin, Inzh.-Fiz. Zh., 26, No. 1, 117-121 (1974).
13. V. G. Chernyak, B. T. Porodnov, and P. E. Suetin, Inzh.-Fiz. Zh., 24, No. 2, 227 (1973).
14. S. K. Loyalka and I. W. Cipolla, Phys. Fluids, 14, 1656 (1971).
15. C. Cercignani and C. D. Pagani, Rarefied Gas Dynamics, Vol. 1, Academic Press, New York (1969), p. 269.
16. V. G. Chernyak, B. T. Porodnov, and P. E. Suetin, Zh. Tekh. Fiz., 43, 2420 (1973).
17. P. E. Suetin, B. T. Porodnov, V. G. Chernyak, and S. F. Borisov, Fluid, Mech., 60, 581 (1973).
18. V. G. Chernyak, B. T. Porodnov, P. E. Suetin, M. V. Nevolin, and V. D. Seleznev, Izv. Vyssh. Uchebn. Zaved., Fiz., No. 6, 107 (1973).
19. P. L. Bhatnager, E. P. Gross, and M. Krook, Phys. Rev., 94, 511 (1954).
20. S. G. Mikhlin, Variational Methods in Mathematical Physics, Macmillan, New York (1964).